MATH 521A: Abstract Algebra Exam 3

Please read the following instructions. For the following exam you are free to use a calculator and any papers you like, but no books or computers. Please turn in **exactly six** problems. You must do problems 1-4, and two more chosen from 5-8. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 75 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 30 and 60. This will then be multiplied by $\frac{5}{3}$ for your exam score.

Turn in problems 1,2,3,4:

1. Factor $f(x) = x^4 + 3x^3 - x^2 + 3x + 1$ into irreducibles in $\mathbb{Z}_5[x]$.
2. Prove that $f(x) = x^3 + 9x^2 + 8x + 96301$ is irreducible in $\mathbb{Q}[x]$.
3. Let R be an integral domain. Prove that all linear polynomials in $R[x]$ are irreducible, if and only if R is a field.
4. Set $f(x) = x^4 + 3x^3 - x^2 + x - 1, g(x) = 2x^5 + 3x^4 + 3x^2 + 2x - 1$, both in $\mathbb{Z}_5[x]$. Use the extended Euclidean algorithm to find $\gcd(f, g)$ and to find polynomials $a(x), b(x)$ such that $\gcd(f(x), g(x)) = a(x)f(x) + b(x)g(x)$.

Turn in exactly two more problems of your choice:

5. Set $f(x) = x^n - x^{n-1} \in F[x]$. Carefully find all divisors of $f(x)$ in $F[x]$.
6. Let $f(x), g(x), h(x) \in F[x]$. Suppose that $f(x)|g(x)h(x)$ and $\gcd(f(x), g(x)) = 1$. Prove that $f(x)|h(x)$.
7. Let p be an odd prime. Prove there is at least one $a \in \mathbb{Z}_p$ such that $x^2 - a$ is irreducible in $\mathbb{Z}_p[x]$.
8. We call a polynomial in $F[x]$ *cinom* if its constant coefficient is 1. Suppose that $f(x)$ is a nonconstant, cinom, polynomial in $F[x]$. Prove that we may write $f(x)$ as the product of irreducible cinom polynomials.

You may also turn in the following (optional):

9. Describe your preferences for your final group assignment. (will be kept confidential)